Abstract

We introduce a new framework for best 2D view selection of 3D models based on the assumption that models belonging to the same class of shapes share the same salient features. The main issue is learning these features. We propose an algorithm for computing these features and their corresponding saliency value. At the learning stage, a large set of features are computed from every model and a boosting algorithm is applied to learn the classification function in the feature space. AdaBoost learns a classifier that relies on a small subset of the features with the mean of weak classifiers, and provides an efficient way for feature selection and combination. Moreover it assigns weights to the selected features which we interpret as a measure of the feature saliency within the class. Our experiments using the LightField (LFD) descriptors and the Princeton Shape Benchmark show the suitability of the approach to 3D shape classification and best-view selection for online visualization of 3D data.

Keywords: 3D Retrieval, 3D Model Classification, Boosting, Best view selection, Feature saliency

1 Introduction

In recent years, with the significant advances in 3D acquisition and modeling, 3D model collections have gained significant importance. They provide a mean for knowledge representation in a wide range of applications including Computer-Aided Design (CAD), molecular biology, virtual archive, and entertainment. However, efficient extraction and reuse of this knowledge depends on the availability of efficient tools for storage, classification, retrieval and visualization of the 3D data. Different approaches have been developed for content-based 3D model classification, recognition and retrieval but none of them has achieved high performance on all types of shape classes. This is because of the negative effects caused by the semantic gap between the lower level visual features and high level semantic concepts [Hou and Ramani 2006].

The goal of this paper is to develop an effective algorithm for the selection of the best views of a 3D model. The proposed algorithm is based on the assumption that the 3D models that belong to the same class of shapes share the same salient features. Therefore, finding the best views of a 3D model, that we call representative feature set, can be regarded as a machine learning task. Particularly, supervised learning of shape features allows to capture the high-level semantic concepts of the data using low-level geometric features.

The approach we propose in this paper is based on boosting. Our key idea is to use a large set of local and global features that describe the shape when viewed from a different viewing angles, then use AdaBoost [Schapire 2003] to select only the most efficient ones. Boosting is a mean for classifier combination, and therefore, it provides an efficient way for feature selection and combination. It has been efficiently used for online learning of the query features for relevance feedback in image retrieval [Tieu and Viola 2004; Amores et al. 2004]. Boosting, like many machine-learning methods, is entirely data-driven in the sense that the classifier it generates is derived exclusively from the evidence present in the training data itself [Schapire 2003]. Moreover, allowing redundancy and overlapping in the feature set has been proven to be more efficient in recognition and classifications tasks than orthogonal features [Tieu and Viola 2004].

The problem of defining representative 2D views of 3D models has received increasing attention in recent years. Early works study the similarity and stability relationship between different 2D views of a 3D model [Denton et al. 2004; Yamauchi et al. 2006]. The common approach is to extract a set of features from the 3D model, quantify the importance of each feature, define the importance of a view as a function of the importance of the features that are visible from a given viewpoint, then select the view that maximizes this quantity. Recent works based on this idea are the mesh saliency [Lee et al. 2005] and the paper of Yamauchi et al. [Yamauchi et al. 2006].

The main issue in the previously proposed algorithms is that the solutions they propose consider isolated 3D models out of context. However, in order to capture the high level semantic concepts of the 3D shapes, which are very important for 3D data visualization and exploration, it is important to consider the problem in the context of 3D shape repository where the data are clustered into classes. The models within each class share common semantic concepts. Therefore, best view selection and view saliency quantification can be casted into the problem of feature selection and feature importance measurement.

Recent progress in pattern recognition and machine learning suggested the use of supervised learning to narrow the semantic gap. This allows the automatic selection of salient features of a single 3D model within a class of shapes, and also the use of the results of classification to improve the performance of retrieval and classification algorithms. The basic learning approach is the Nearest Neighbor classification. It has been used for the classification of 3D protein databases [Ankerst et al. 1999], and also to the classification of 3D engineering parts [Ip et al. 2003].

Hou et al. [Hou et al. 2005] introduced a semi-supervised semantic clustering method based on Support Vector Machines (SVM) to organize 3D models semantically. SVM have been widely used in statistical learning. The given query model is first labeled with some semantic concepts such that it can be assigned to a single cluster. Then the search is conducted only in the corresponding cluster. Supervised learning and ground-truth data are used to learn the patterns of each semantic cluster off-line. Later, they extend this work [Hou and Ramani 2006] to combine both semantic concepts and visual content in a unified framework using probability-based classifier. They use a linear combination of several classifiers, one classifier per shape descriptor. The individual classifiers, which are trained in a supervised manner, output an estimate of the probability of data being classified to a specific class. It uses also the output of the training stage to estimate the optimal weights of the combination model. In this approach features to use and type of classifiers are set manually. The method we propose provides a framework for automatic feature selection and weight assignment.

Shilane et al. [Shilane and Funkhouser 2006; Funkhouser and Shi-
At the run-time, given the user-specified 3D model combined into one multi-class classifier. Finally, the binary classifiers are trained using a learning approach to select the most important views of 3D models. The selected views are consistent for all objects of the same class, and are suitable for multi-scale organization of the shape space based on the hierarchical classification of the training set.

This paper is organized as follows: Section 2 gives and overview of the proposed framework and outlines the main contributions. Section 3 details the feature selection and combination algorithm for binary classification problems. The generalization to a multi-class problem, and to unseen 3D models are presented in Section 4.1 and 4.2. Experimental results are provided in Section 5. Section 6 concludes the paper.

2 Overview of the approach

Our approach performs as follows; During the training stage a strong classifier is learned using AdaBoost. The classifier returns the likelihood that a given 3D model \( O \) belongs to a class of shapes \( C \). First a large set of features are extracted. In our implementation we used 100 Light Field Descriptors (LFD) [Chen et al. 2003]. Each Light Field descriptor encodes the properties of a 2D projection of a 3D shape. Then a set of binary classifiers are trained using AdaBoost. Each binary classifier learns one class of shapes and its optimal set of salient views. Finally, the binary classifiers are combined into one multi-class classifier.

At the run-time, given the user-specified 3D model \( Q \), a ranked list of \( k \)-best views is produced in a two-stage process. First, a large set of features are computed from the query model \( Q \), in the same manner as for the database models. Then in the first stage, a set of highly relevant classes to \( Q \) is found. Each binary classifier \( C_i \) decides whether the class \( C_i \) is relevant to the query \( Q \) or not. The class with highest posterior probability \( C_Q = \arg\max_{C} P(C|Q) \) is selected. In the final stage, the best views of the query model \( Q \) are the selected views of the class of shapes \( C_Q \).

The key step is the way we predict the saliency of each feature with respect to a class of shapes in the training set. More formally, the saliency of a feature \( v \) with respect to a class of shapes \( C \) is the ability of this feature to discriminate the shapes of class \( C \) from the shapes of other classes in the database. Mathematically, given the binary classifier \( C \) trained with the feature \( v \), the saliency of \( v \) is directly related to the overall classification error of \( C \) on the data set. However, none of the existing classifiers that are based on a single feature can achieve zero classification error. Therefore none of the features is sufficiently salient. AdaBoost provides a way for combining weak classifiers and shape features with different saliency degrees, into a single strong classifier with high classification performance. There are several advantages of this approach:

- Although a large set of features is extracted both at the training and online stages, only a small subset of the features (between 10 to 50) is used during the similarity estimation. This allows retrieval at interactive rates.
- The algorithm selects automatically the representative set of features for each class of shapes, and provides a mean for automatic combination of the selected features. This has potential applications in shape retrieval and recognition.

For feature extraction, we use the Light Field descriptors (LFD) proposed by Chen et al. [Chen et al. 2003], which has been proven to be the most effective on the Princeton Shape Benchmark (PSB) [Shilane et al. 2004]. However, a further investigation is required to test the efficiency of other 2D view descriptors when boosted.

3 Supervised classification - the binary case

The first task in our approach is to build a classifier \( C \) that decides whether a given 3D model \( O \) belongs to a class of shapes \( C \) or not. The challenge is to define a feature space such that 3D shapes belonging to the same class are mapped into points close to each other in the new feature space. Clusters in this feature space will correspond to classes of 3D models. There are many feature spaces that have been proposed in the literature, but it has been proven that none of them achieved best performance on all classes. We propose to follow a machine learning approach where each classifier is obtained by the mean of training data. In the following we explain in detail each step in the case of a binary classification problem.

3.1 Feature extraction

The process starts by computing a large set of features for each model in the training set, the contents of the database to search. There are many requirements that the features should fulfill: (1) compactness, (2) computation speed, and (3) the ability to discriminate between dissimilar shapes. However, in real applications it is hard to fulfill these requirements when the goal is to achieve high retrieval accuracy. In fact, compact features, which are easy to compute, are not discriminative enough to be used for high accuracy retrieval. We propose to extract a large set of features following the same idea as in [Tieu and Viola 2004].

There are many shape descriptors that can be computed from a 3D model. A large set of spherical harmonics [Funkhouser and Shilane 2006] and spherical wavelet-based descriptors [Laga et al. 2006] can be computed by moving the center of the sphere across different locations on the shape’s surface or on a 3D grid. However, in the literature, it has been proven that view-based descriptors outperform significantly the spherical descriptors. We propose to use the Light field descriptors (LFD).

First, all the models in the database are translated to their center of mass, scaled to fit inside a unit sphere, and normalized for rotation using continuous PCA [Varanic 2003]. Then we compute for each 3D model a set of 100 Light Field descriptors in the same manner as in [Chen et al. 2003]. Recall that the length of one light field descriptor is 45. Therefore, every 3D model is represented with a set of 100 vectors of dimension 45. Each LFD provides a description of the shape when viewed from the corresponding projection point.

3.2 Boosting the binary classification

A brute force approach for comparing a large set of features is computationally very expensive. In the best case, it requires \( M \times d \times N \)
comparisons, where $M$ is the number of feature vectors used to describe a 3D model, $d$ is the dimension of the feature space, and $N$ is the number of models in the database.

Previous work consider this problem from the dimensionality reduction point of view. Ohbuchi et al. [Ohbuchi et al. 2007] provides an overview and performance evaluation of six linear and non-linear dimensionality reduction techniques in the context of 3D model retrieval and demonstrated that non-linear techniques improve significantly the retrieval performance. There have been also a lot of research in classifiers that have a good generalization performance by maximizing the margin. The major advantage of boosting over other classification algorithms such as Support Vector Machines (SVM) [Hou et al. 2005], and non-linear dimensionality reduction techniques [Ohbuchi et al. 2007; Ohbuchi and Kobayashi 2006] is its speediness. Moreover, it provides a good theoretical and practical quantification of the upper bound of the error rate, therefore a good generalization performance. Furthermore, it can be used as a feature selection algorithm.

We use AdaBoost version of boosting. Every weak classifier is based on a single feature of a 3D shape (recall that we have computed a large set of features for each 3D model). The final strong classifier, a weighted sum of weak classifiers, is based on the most discriminant features weighted by their discriminant power. The algorithm is summarized in Algorithm 1. The output of the strong classifier can be interpreted as the posterior probability of a class $C$ given the shape $O$:

$$P(C|O) = \frac{e^{f_C(O)}}{e^{f_C(O)} + e^{-f_C(O)}}$$  \hspace{1cm} (1)$$

where $f_C(O)$ is the weighted average of the base classifiers produced by AdaBoost for the 3D object $O$.

AdaBoost requires only two parameters to tune; the type of weak classifier, and the maximum number of iterations, i.e., the number of weak classifiers. The classification performance of the weak classifier is only required to be slightly better than random. We used the LMS classifier because of its simplicity. The parameter $T$ can be set such that $E[f_C]$, the upper bound of the classification error on the training data of the strong classifier $f_C$, is less than a threshold $\theta$. In our experiments we found that a value of $T$ between 20 and 50 is sufficient to achieve an upper bound of the classification error on the training set less than 1.0%.

For training the classifiers we use as positive and negative examples the relevant and non-relevant models provided in the Princeton Shape Benchmark (PSB) classification. For example, to build a strong classifier that learns the decision boundary between the $biped$ human objects and $non$-biped human objects, the positive examples are set to all models that belong to the class $biped$ human, while the negative examples are the remaining models in the database. The PSB is provided with a train and test classifications. We use the train classification to train our classification and the test classification to assess the performance of the classification and retrieval.

Interpretation of the weak classifiers

Boosting algorithm can be used as a feature selection and combination technique. Each iteration learns a new weak classifier that is based on the most discriminative feature according to the probability distribution of the training data. In the case of LFD, the selected feature is the descriptor of a 2D projection of a 3D model. Therefore, by adopting a Boosting approach we provide a tool for best view selection and view ordering based on their ability to discriminate the shapes of a certain class from the other classes in the database. Recall that here we assume that the quality of a view is quantified as its discrimination ability. Furthermore, the weight of each weak classifier can be considered as a measure of the saliency of the selected feature.

### 4 Generalization

#### 4.1 Generalization to multiple classes

Two straightforward extensions schemes are the one-vs-all classifier and the pairwise classifier [Hao and Luo 2006]. The pairwise classifier uses $L(L-1)/2$ binary classifiers, where $L$ is the number of classes in the training set, to separate each class from the other classes. A voting scheme at the end is used to determine the correct classification [Hao and Luo 2006]. With the one-vs-all classifier, $L$ AdaBoost-based binary classifiers are trained, each of which is able to distinguish one class from all the others. The pairwise classifier has a smaller area of confusion in the feature space compared to the one-vs-all. In our implementation we used a one-vs-all classifier for its simplicity. The details of the algorithm are sketched in Algorithm 2.

The output of the training stage is a set of $L$ binary classifiers, where $L$ is the number of classes in the database. Given a query model $Q$ each binary classifier will return a vote for a certain class. We use the positive votes to construct the set of candidate classes to which the query $Q$ may belong. It is important to notice that when a new 3D model or a new class of models are added to the database, only the classifier that corresponds to the model’s class that needs training.

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**Algorithm 1: AdaBoost algorithm for binary classification**

**Input:**
- Training set $S_C = \{(V_i, y_i), i = 1 \ldots N\}$, where $V_i = \{v_1, \ldots, v_K\}$ a large set of $K$ features computed from the 3D object $O_i$, $y_i \in \{+1, -1\}$ the desired classification of $O_i$.

**Output:**
- The decision function $f_C$, such that, $f_C(O) > 0$ is $O \in C$, and $f_C(O) < 0$ if $O \notin C$.

1. Initialize the sample weights: $w_i$ for $i = 1, \ldots, N$:

   $$w_i = \begin{cases} \frac{1}{N}, & \text{if } O_i \text{ is a positive example} \\ \frac{1}{N}, & \text{otherwise.} \end{cases}$$

   where $N^+$ and $N^-$ are, respectively, the number of positive and negative examples.

2. for $t = 1, \ldots, T$ do
   a. Choose the hypothesis $h_t$ with the lowest classification error $\epsilon_t$.
   b. Update the sample weights:

   $$w_{i+1} = \frac{w_i e^{-\alpha h_t(O_i)y_i}}{Z_t}$$

   where $O_t$ is correctly or incorrectly classified by the weak hypothesis $h_t$, $\alpha_t = 0.5 \log \frac{1-\epsilon_t}{\epsilon_t}$, and $Z_t$ is a normalizing constant so that $w_{i+1}$ is a distribution.

3. Final classifier: $f_C(O) = \sum_{t=1}^{T} \alpha_t h_t(O)$. 

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**Algorithm 2: AdaBoost algorithm for pairwise classification**

**Input:**
- Training set $S_C = \{(V_i, y_i), i = 1 \ldots N\}$, where $V_i = \{v_1, \ldots, v_K\}$ a large set of $K$ features computed from the 3D object $O_i$, $y_i \in \{+1, -1\}$ the desired classification of $O_i$.

**Output:**
- The decision function $f_C$, such that, $f_C(O) > 0$ is $O \in C$, and $f_C(O) < 0$ if $O \notin C$.

1. Initialize the sample weights: $w_i$ for $i = 1, \ldots, N$:

   $$w_i = \begin{cases} \frac{1}{N}, & \text{if } O_i \text{ is a positive example} \\ \frac{1}{N}, & \text{otherwise.} \end{cases}$$

   where $N^+$ and $N^-$ are, respectively, the number of positive and negative examples.

2. for $t = 1, \ldots, T$ do
   a. Choose the hypothesis $h_t$ with the lowest classification error $\epsilon_t$.
   b. Update the sample weights:

   $$w_{i+1} = \frac{w_i e^{-\alpha h_t(O_i)y_i}}{Z_t}$$

   where $O_t$ is correctly or incorrectly classified by the weak hypothesis $h_t$, $\alpha_t = 0.5 \log \frac{1-\epsilon_t}{\epsilon_t}$, and $Z_t$ is a normalizing constant so that $w_{i+1}$ is a distribution.

3. Final classifier: $f_C(O) = \sum_{t=1}^{T} \alpha_t h_t(O)$.
Algorithm 2: One-vs-all extension of binary AdaBoost for multi-class problem.

**Input:**
- Training set $S_l = \{(V_i^l, y_i^l), i = 1, \ldots, N\}, l = 1, \ldots, L$, where $V_i^l = \{v_1^l, \ldots, v_K^l\}$ a large set of $K$ features computed from the 3D object $O_i$, $y_i^l \in \{+1, -1\}$ the desired classification of $O_i$.

**Output:**
- $L$ binary decision functions $f_{C_l}$, such that, $f_{C_l}(O) > 0$ is $C_l$ is a candidate class for the 3D model $O$, and $f_{C_l}(O) < 0$ otherwise.

for $l = 1, \ldots, L$ do
  1. Train one strong binary classifier $\mathcal{C}_l$, using Algorithm 1.
  2. Train $f_{C_l}(O) > 0$ if $O \in C_l$, and negative otherwise.
end

Final classifier: $\mathcal{C} = \{\mathcal{C}_l, l = 1, \ldots, L\}$.

4.2 Generalization to unseen 3D models

At the run time, the user specifies a 3D model, that we call a query $Q$, and seeks to find its salient 2D views. This is performed in two steps: first we seek to find the candidate classes to which the query may belong. Then, the best views of the query model are those selected for its best candidate class.

To classify the query $Q$, we compute a set of $M$ feature vectors in (LFD descriptors in our case) in the same manner as in the training stage (Section 3.1). Then we let each binary classifier $\mathcal{C}_l$ vote for a class $C_l, l = 1, \ldots, L$. The candidate classes are determined by the classifiers that have positive response to the query $Q$. We order them in descending order of the class posterior probabilities given in Equation 1. Next, we select the class with the highest response and assign to the 3D model the best views that have been learned for this class, i.e., the salient features of the class $C_l$. Notice that the classification is performed only on a subset of the large set of features. This has significant impact on the computation time.

5 Experimental results

To evaluate the performance of the proposed approach, we use the Princeton Shape Benchmark (PSB) [Shilane and Funkhouser 2006] training and test sets, and the Shape Retrieval Evaluation Contest (SHREC2006) [Veltkamp et al. 2006] query set and performance evaluation tools. The Princeton Shape Benchmark contains 1814 polygon soup models, divided into the training set (907 models) and the test set (907 models). Every set contains four classification levels; the base train classification contains 129 classes while the coarsest classification (coarse3) contains two classes: man-made and natural objects. We use the base train classification to train our classifiers and the test set to assess the classification performance.

Figure 1 shows three models from three different classes (aircraft, airplane, biplane, quadrangle, animal, feline, and vehicle, car, sports car) and their selected 2D views ordered from the highest saliency value to the lowest. This shows first that the selected views are consistent across all models of a same class, and are visually plausible. This is because boosting captures some high semantic features of the data set.

To evaluate quantitatively the efficiency of the best view selection algorithm, we assume that the selected views are good enough if they achieve good classification and retrieval performance when they are used to index 3D model collections. Figure 2 summarizes the classification performance of the developed AdaBoost classifier. In this figure, the average classification performance is the ratio between the number of correctly classified models of a class $C$ to the total number of models in the class. We see that, for the coarse3 classification (Figure 2(d)), which contains only two classes with very high shape variability within each class, the classification performance is at 65.3% for natural shape and 73% for man-made models. This clearly proves that the training procedure captures efficiently the semantic concepts of the shape classes and generalizes relatively well to unseen samples.

The performance on the other classification levels: base, coarse1 and coarse2 are shown in Figure 2(a), (b) and (c). In this experiment we show only the classification results on the classes of the test set that exist in the training set. On the base classification (Figure 2(a)), we can see that the classifiers achieve 100% classification performance on space, ship, enterprise, like, dining, chair and sea, vessel. The worst performance in on the plant, tree models. This is because probably the class has high variability and many small detailed features that cannot be captured by the Light Field descriptors.

Finally, to evaluate the retrieval performance we use the query set of the SHREC2006. Recall that none of the query models is present in the database. Therefore, they can be used to assess the ability of the classifier to generalize to unseen models. We compare with the algorithms that have been benchmarked in the contest [Veltkamp et al. 2006]. We show only the top six results but the reader can refer to [Veltkamp et al. 2006] for a complete comparison. Each query has a set of highly relevant classes, relevant classes, and not relevant classes.

Table 1 summarizes the performance on the Mean Average Precision, Mean First Tier and Second Tier, for both highly relevant and relevant classes. Our method ranks top on all measures for relevant classes. Moreover, it outperforms significantly the other methods on the Mean Second Tier for both highly relevant and relevant classes. This shows that the combination of classification and search improves the ability to retrieve the relevant results in the top of the retrieved list. Our method however, achieved relatively low performance on Cumulative Gain-related performance measures.

We believe that this is because of lack of data at the training stage and therefore, it is hard to capture the salient features of the class. We plan in the future to experiment with larger databases.

6 Conclusion

We have proposed in this paper a new framework for best view selection of 3D models. By using a boosting approach we are able to use a large set of features in order to capture the high level semantic concepts of different shape classes. Moreover, we provide a way to quantify the saliency of a 2D view with respect to the classification. The developed algorithm allows to use simultaneously a cascade of shaped descriptors. Although we have experimented only with one type of descriptors, we may want to use a different set of descriptors for classification.

This work opens many avenues to explore. First, the framework we proposed allows the use of heterogeneous features, all what we need is to plug new types of descriptors to the training process. Also we plan to investigate on the meaning of the selected feature space for each shape class and extend the framework to the problem of building creative prototypes of 3D object classes, where the prototype should capture the high level semantic features of the class.
Figure 1: The first six views selected by the Boosting algorithm and ordered by the decreased saliency value. Top: aircraft, airplane, biplane, middle: quadruple animal, feline, and bottom: vehicle, car, sports car.

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(a) Mean Average Precision (highly relevant).

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(b) Mean Average precision (Relevant).

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(c) Mean First Tier (Highly relevant).

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(d) Mean First Tier (Relevant).

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(e) Mean Second Tier (Highly Relevant).

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<td>Our method</td>
<td>42.73%</td>
</tr>
<tr>
<td>2</td>
<td>Shilane et al. (R2)</td>
<td>26.58%</td>
</tr>
<tr>
<td>3</td>
<td>Shilane et al. (R3)</td>
<td>26.26%</td>
</tr>
<tr>
<td>4</td>
<td>Makadia et al. (R2)</td>
<td>25.22%</td>
</tr>
<tr>
<td>5</td>
<td>Zaharia et al. (R1)</td>
<td>24.63%</td>
</tr>
<tr>
<td>6</td>
<td>Papadakis et al. (R1)</td>
<td>24.24%</td>
</tr>
</tbody>
</table>

(f) Mean Second Tier (Relevant).

Table 1: Mean Average precision, Mean First Tier and Second Tier performance.

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References

Figure 2: Average classification performance for each class of shapes in the test set of the Princeton Shape Benchmark.


