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Circular-Parade Illusion Created by Half-Body Objects and Mirrors

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Abstract
This paper presents a new visual phenomenon in which the shape of a whole animal is created by optical illusion when a physical 3D object representing a head half or a tail half is placed in contact with a mirror, and the resulting whole animal makes a U-turn in another mirror that is perpendicular to the first mirror. Using two such half-body objects, we can create a circular parade of four animals that orient clockwise or counterclockwise uniformly. The mathematics and design principles of this phenomenon are shown with examples. Some extensions are also discussed.

1 Introduction

The history of visual art is the concatenation of endless efforts to invent methodologies for creating new visual effects [1]. For example, perspective projection techniques enable us to draw object shapes as we see them directly, impressionism places a higher priority on subjective impressions than on correct appearances, cubism aggregates different viewpoints into one picture, and op. art uses optical illusion directly to create visual effects.

Impossible objects also are used as tools to create new visual effects. The term "impossible object" was first used to represent an imaginary 3D structure that is evoked in our mind when we see a special kind of picture called an "impossible figure" [2, 3, 4, 5]. It is well known that Dutch artist M. C. Escher used impossible objects for his wood-cut artworks [6, 7]. Impossible objects are also studied in visual psychology because they generate optical illusions that give us an impression of a 3D structure, but at the same time, we feel that that structure cannot exist [8, 9].

Soon after, several techniques were found for creating real 3D models whose appearances match the impossible figures. One trick is to use curved surfaces that look planar [10], another trick is to use discontinuous gaps that look connected when seen from a certain viewpoint [8, 11], and still another trick is to use non-rectangular angles that look to be 90 degrees [12]. Therefore, the term "impossible object" is also used to represent a real 3D object whose appearance and behavior look impossible when seen from a special viewpoint.

In 2014, we found a new class of impossible objects in which a real 3D object changes its shape so drastically in a mirror that we feel that it is another object, rather than a different view of the same object. We named this phenomenon the "ambiguous cylinder illusion," because the object is cylindrical in its shape [13]. The principle of ambiguous cylinders has been extended in various directions; examples are "topology-disturbing objects" that change ways of connection in a mirror, "partly invisible objects," part of which becomes invisible in a mirror, and "ambiguous tiling," in which a tiling pattern changes to another tiling pattern in a mirror [14].

Another direction of the extension of the ambiguous cylinders is the change of the object's posture instead of its shape. Examples are "left-right reversing objects," in which an object facing toward the right appears to make a U-turn so that it faces toward the left when viewed in a mirror [15], "translation objects," in which an object facing toward a mirror translates into the mirror instead of making a U-turn [16], and "translation-rotation objects," in which an object translates into a front mirror and makes a U-turn in a side mirror [17].

In the present paper, we propose a further extension in this direction. The proposed object represents a head half or a tail half of an animal, but when it is positioned next to a mirror, the corresponding other half appears in the mirror, resulting in the whole shape of the animal. Moreover, this whole shape makes a U-turn in another mirror that is perpendicular to the first mirror. Using two such half-body objects, we can make a circular sequence of four animals all facing in the same orientation, i.e., clockwise or counterclockwise.

We review the translation-rotation objects (Section 2), construct our new object by dividing the translation-rotation object into halves (Section 3), and discuss its variants (Section 4) before concluding our results (Section 5).

2 Review of the Translation-Rotation Illusion

We briefly review the translation-rotation illusion in this section, and then modify the objects involved to create our new illusion in the next section. Figure 1 shows the plan view (the view seen from above) of the setting for the translation-rotation illusion. The xyz coordinate system is fixed in such a way that the xy plane is horizontal. Then, a mirror $\mathrm{M_R}$ is placed vertically on the yz plane, and another mirror $\mathrm{M_L}$ is placed on the xz plane so that they meet along the z-axis at a right angle. An object S is placed on the xy plane. Suppose that the object S has a specific line through it named the "head-tail axis" (why we use this name will be clear soon); in Figure 1 the head-tail axis is presented by the dot-dash line. The object is oriented with the head-tail axis being parallel to the left mirror M_L and perpendicular to the right mirror M_R. This scene is intended to be viewed horizontally from an angle of 45 degrees from both of the mirror surfaces, as represented by the arrow v. The mirror M_R creates the right mirror image S_1, the mirror M_L creates the left mirror image S_2, and still another image S_3 is created by the reflections with the two mirrors.

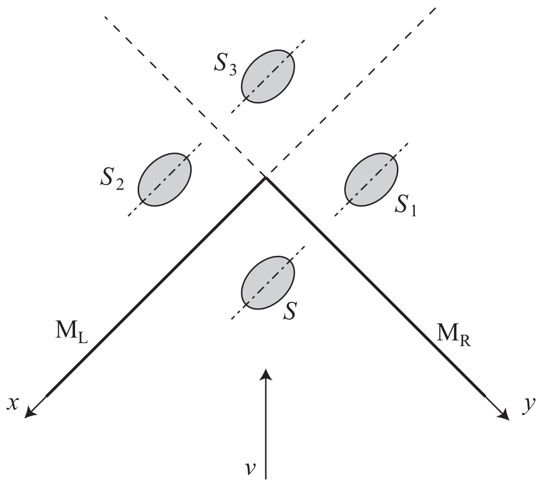

Figure 1: Setting for the translation-rotation illusion.

Figure 2 shows an example of the translation-rotation illusion. There are four dogs. The second dog from the left is the real 3D object, which shows a standing dog facing in a direction opposite to the right mirror M_R, and consequently in parallel with the left mirror M_L. The rightmost dog is the mirror image reflected by M_R, which is perpendicular to the direction the dog is facing. However, the original dog appears to be translated into the right mirror instead of making a U-turn as might be expected. Thus this object creates the translation illusion. The leftmost dog is the reflection by the mirror M_L. However, in this case, the dog makes a U-turn (i.e., a rotation by 180 degrees) in the mirror instead of translating into the mirror. Thus the left-right reversal illusion occurs in the left mirror. Since the object creates two different illusions in the two mirrors, we call this visual phenomenon the "translation-rotation illusion."

Note that, in Figure 1, the object S and its mirror image S_3 are placed almost on the same line of sight, and hence S_3 would be occluded by S. In order to avoid the occlusion in Figure 2, we shift the object S toward the left mirror so that all the four objects can be seen without occlusion.

Note, further, that the viewpoint in Figure 2 is a little higher than the object, so that the scene is seen from above in a slanted direction. We choose this viewpoint because we can see the bottom lines of the mirrors, and we can understand which is the real object and which are its reflections.

The readers may intuit the naming of the "head-tail axis" from this example; this is the axis that connects the head part of the dog to the tail part. Note, however, that it is impossible to specify which is the head and which is the tail. Each part looks like the head or the tail, depending on the viewing direction. Refer to [17] for the method for designing this class of objects.

Figure 2: translation-rotation illusion created by a dog object.

An important property of this kind of objects is that they are plane-symmetric with respect to a plane perpendicular to the head-tail axis. Figure 3 shows three-view CG images of the 3D dog model. The top left is the plan view, the bottom left is the front view, the bottom right is the side view, and the top right is the appearance of the object seen along the same direction as that in Figure 2. The top view and the front view show that this object is plane symmetric, with the plane of symmetry represented by the red lines. In Figure 2, the object is placed in such a way that the plane of symmetry is parallel to the right mirror $\mathrm{M_R}$ and perpendicular to the left mirror $\mathrm{M_L}$.

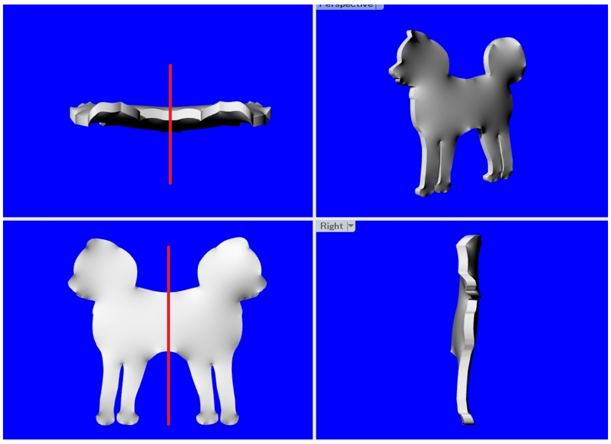

Figure 3: Three-view diagram of the dog object; red lines represent the plane of symmetry.

We may intuitively understand the reason why the translation-rotation illusion occurs in the following manner. Let $c_x(t)$, $c_y(t)$, and $c_z(t)$ be real-valued continuous functions of the parameter t ($0 \le t \le 1$) such that $c_x(0) = c_x(1)$, $c_y(0) = c_y(1)$, and $c_z(0) = c_z(1)$. Then,

$$C = \{(c_x(t), c_y(t), c_z(t)) \mid 0 \le t \le 1\} \quad (1)$$

represents a closed curve in the 3D space. As shown in Figure 4, we can fix the curve C in front of the two mirrors $\mathrm{M_R}$ and $\mathrm{M_L}$. Let P be the orthographically projected picture of the space curve C that we obtain when we see the curve from the direction v. Then, let Q_R and Q_L be the orthographically projected pictures of C that we obtain when we see the curve from the right and from the left, respectively, in the direction perpendicular to v. Finally, let P_R and P_L be the orthographically projected pictures of the right mirror image and the left mirror image, respectively, of the curve C.

For any 2D picture S, we denote by $\Gamma(S)$ the picture that is obtained when we exchange the right and the left of S. Since Q_R and Q_L are the pictures of the same curve seen from mutually opposite directions, we get

$$Q_L = \Gamma(Q_R). \quad (2)$$

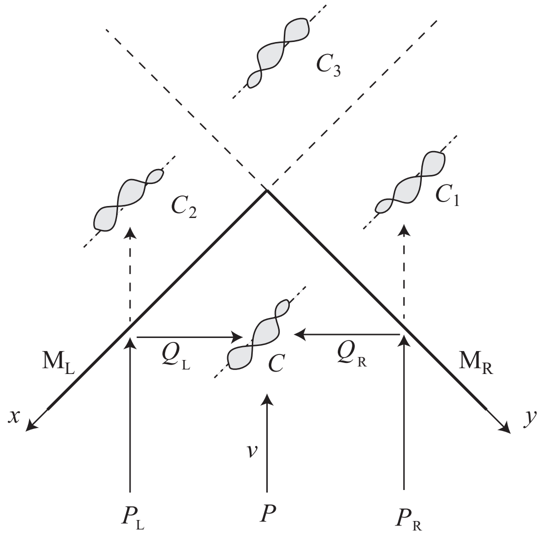

Figure 4: Direct appearance of a space curve and its two mirror images.

Because P_R and P_L are the mirror images of the curve C reflected by the right and left mirrors, respectively, we get

$$P_R = \Gamma(Q_R) \text{ and } P_L = \Gamma(Q_L). \quad (3)$$

Equations (2) and (3) are valid for any space curve C.

Now, assume that the space curve C is plane symmetric with respect to a plane parallel to the right mirror $\mathrm{M_R}$. Then, we get

$$Q_R = \Gamma(P), \quad (4)$$

and consequently from eqs. (2) and (3), we get

$$P_R = \Gamma(Q_R) = \Gamma(\Gamma(P)) = P, \quad (5)$$
$$P_L = \Gamma(Q_L) = \Gamma(\Gamma(Q_R)) = \Gamma(P). \quad (6)$$

Equation (5) implies that the translation illusion occurs in the right mirror, and eq. (6) implies that the left-right reversal illusion occurs in the left mirror. Thus, any plane-symmetric space curve can create the translation-rotation illusion.

Let us go back to our dog object in Figures 2 and 3. It is not a simple curve but a solid model whose volume is filled with material. Let C be the space curve formed by the silhouette of the object when it is seen from the direction v. Note that all the points on this silhouette curve are visible when we see it from the right direction perpendicular to v; that is, no occlusion occurs. Therefore, the silhouette curve obeys eqs. (2), . . . , (6).

Any 3D object and its mirror image are plane symmetric with respect to the mirror surface. Because the dog object is plane symmetric and the plane of symmetry is parallel to the mirror M_R, the mirror image of the object coincides with the translated version of the object. However, when we see this object from the viewing direction shown in Figure 2, one part appears to be the head and the other part the tail; it is thus difficult to recognize the plane symmetry and consequently we perceive that the object translates into the right mirror instead of making a U-turn.

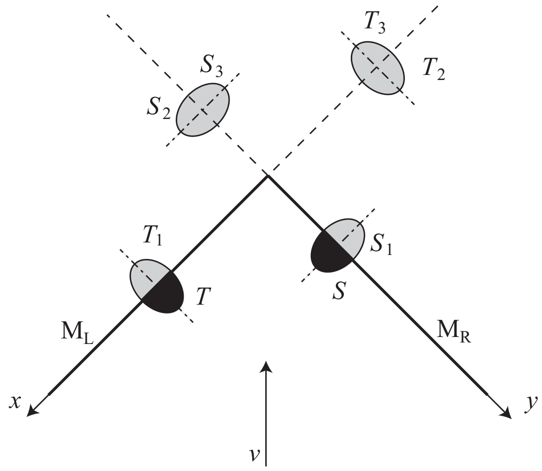

Figure 5: Setting for the circular-parade illusion created by two half-body objects S and T.

3 Circular-Parade Illusion Created by Half-Body Objects

Once we have the translation-rotation object, we can construct a new illusion by cutting the object into two halves along its plane of symmetry. The setting of the new illusion is presented in Figure 5, where two vertical mirrors M_R and M_L are placed in the same way as in Figure 1. Black areas represent real physical objects, and gray areas represent their mirror images. A half-body object, represented by S, is place in such a way the cut plane coincides with the right mirror surface M_R, and consequently it generates the mirror image S_1. Additionally, the pair of S and S_1 generates another pair of mirror images S_2 and S_3 in the other mirror M_L. Another half-body object, represented by T, is placed so that the cut plane coincides with the left mirror surface M_L, and it creates the mirror image T_1 in M_L, and the other pair of images T_2 and T_3 are created in M_R.

Suppose that the real 3D object in Figure 2 (the second object from the left) is cut along the plane of symmetry into halves, and the head half (the left half) is moved without changing its orientation to the position S in Figure 5 so that the cut plane touches the mirror M_R. Figure 6 shows a scene in which the head-half object indicated by the arrow is placed very close to the right mirror. It does not touch the mirror and thus we can clearly see that it is only a half-body object. Then, this object creates the mirror image S_1, which appears to be a tail, and the resulting pair of S and S_1 gives the whole dog, which is the same as the original dog object in Figure 2. The other mirror M_L creates the pair of mirror images S_2 and S_3, which appear to be the same dog as the leftmost dog in Figure 2. Note that the half-body object S and its image S_1 form an object that is plane symmetric with respect to the mirror surface, and hence the equivalence of eq. (6) can be applied to this object. Thus, the U-turn property is guaranteed in the left mirror.

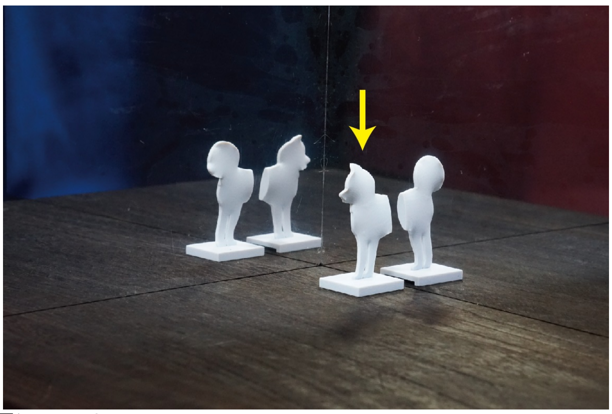
Figure 6: Dog-head object and its mirror images; the head half indicated by the arrow is a real 3D object and the others are its mirror images.

Next, we place another copy of the same half-body object at the position T in Figure 5. This situation is shown in Figure 7, where the real half-body object indicated by the arrow is placed very close to the left mirror M_{L}. In this orientation, this half-body object appears to be a tail half, while the mirror image T_1 in the left mirror M_{L} creates the head half. This head-tail pair together creates another pair of mirror images T_2 and T_3 to form the other dog facing toward the opposite direction in M_{R}.

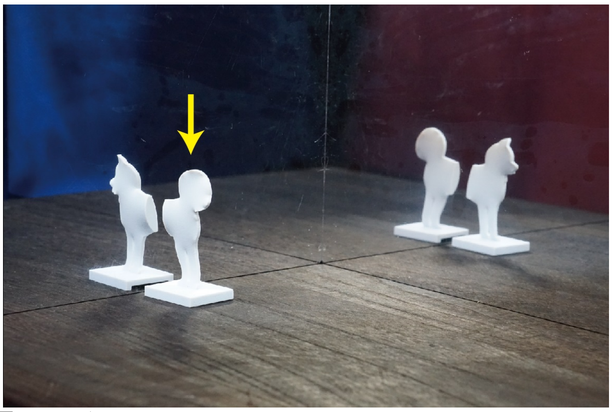

Figure 7: Dog-tail object and its mirror images; the tail half indicated by the arrow is a real 3D object and the others are its mirror images.

Combining the two half-body objects in Figures 6 and 7, we can create the circular parade of four dogs as shown in Figure 8

Figure 8: Circular parade of four dogs created by the two half-body objects shown in Figures 6 and 7.

The process for creating the new illusion shown by the above example can be summarized by the following. We first choose a 3D real object that creates the translation-rotation illusion. This object is plane symmetric. We next cut the object along its plane of symmetry into two halves. Finally we choose one of the two halves, make two copies of it, and place them at S and T in Figure 5. This produces a circular parade of four objects with the same orientation. The orientation depends on which half we choose. One of the two half-body objects creates a clockwise parade, and the other a counterclockwise parade.

Two more examples of the new illusion created in this method are shown in Figures 9 and 10. In Figure 9, four bears walk clockwise. Real 3D objects are the tail half of the leftmost bear and the head half of the third bear from the left; all other parts are their mirror images.

In Figure 10, four squirrels face clockwise along a circle. Real 3D objects are the tail half of the leftmost squirrel and the head half of the third squirrel from the left; all other parts are their mirror images.

Note that the ideal viewing direction for this illusion is along the horizontal, but in both figures, the viewing direction is slanted slightly downward, yet the illusion is still maintained. Actually, this illusion is robust in the sense that the allowed range of the viewing direction is relatively large, that is, the illusion does not disappear even when we observe the scene from a slightly high viewpoint as shown in these figures. This is one of remarkable features of the ambiguous cylinders and their extensions; in contrast, the classical impossible objects designed to realize impossible figures work only when they are seen from very specific viewpoints.

Figure 9: Circular parade of walking bears.

Figure 10: Circular parade of squirrels.

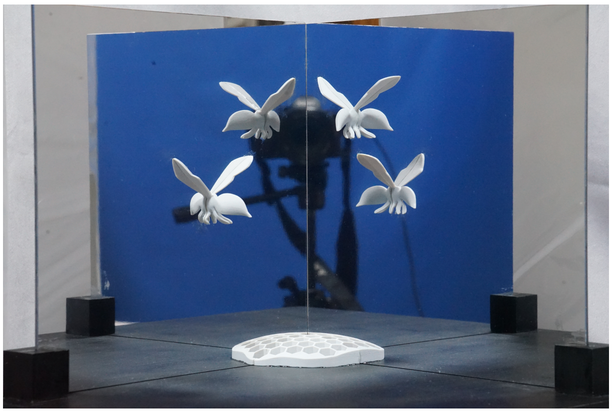

Figure 11: Circular flight of bees.

4 Variations

The illusion just described can be extended in several directions. First, because the half-body object is placed against the mirror's surface, we can attach it directly to the surface and consequently create an image of an object floating in space rather than standing on a horizontal surface. An example is shown in Figure 11, where four bees are flying clockwise around a beehive. The mirror reflection makes the images darker than the original objects because the light-intensity reflection rate is less than 1. Therefore, we can distinguish between the original half-bodies, which are brighter, and their mirror images, which are darker. In Figure 11, the real physical objects are the tail part of the leftmost bee (i.e., the lower left bee) and the head part of the third bee from the left (i.e., the upper right bee). The beehive is created by a real object representing a quarter and its mirror images.

Note that the camera itself is reflected by the mirrors. In other figures we have seen so far, the camera does not appear. This is because the camera is placed in a dark position, or the camera is out of sight due to the slanted viewing direction.

By gluing two or more half-objects onto each mirror surface, we can create a larger flock of creatures. Figure 12 shows eight birds flying clockwise, four of which are in the upper level and the other four in the lower level. In this example, we use four real 3D half-bodies. The readers might be able to recognize them because they are brighter than the other parts. Indeed, the real 3D objects are the tail of the leftmost bird in the upper level, the tail of the second bird from the left in the lower level, the head of the rightmost bird in the lower level, and the head of the second bird from the right in the upper level.

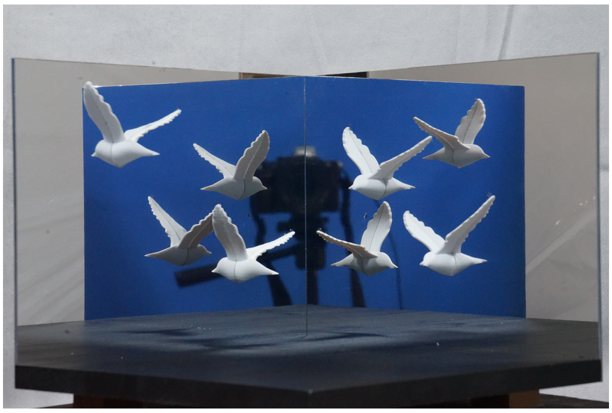

Figure 12: Circular flight of birds.

We can implement the illusion still in another way. Figure 13 shows an example in which two half-body objects and two left-right reversing objects are mixed together with one vertical mirror to form a circular line of six lizards, all facing counterclockwise. The leftmost and the rightmost lizards are created by half-bodies, and the middle four lizards are created by two left-right reversing lizards.

5 Concluding Remarks

We have shown that a circular parade of four objects styled after living creatures can be created

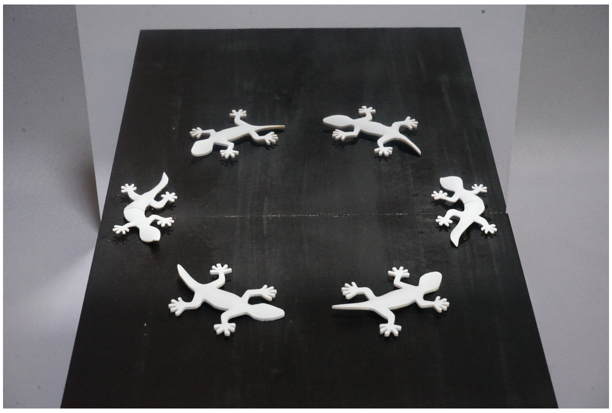

Figure 13: Circular walk of lizards.

by an optical illusion using two half-body objects and two mirrors. The half-body objects are real 3D solid objects, but their behaviors appear impossible because they seem to defy the mirror-reflection rule. Hence, this class of optical illusion also belongs to the impossible objects. It should be noted that the illusion occurs over a relatively wide range of viewpoints, whereas classical impossible objects create the sense of impossibility only when they are seen from a special viewpoint. In the latter case, this viewpoint is strict; the real shape of the object is easily revealed when we shift our viewpoint even slightly, or when we observe the object with both eyes. Seeing the object from a special viewpoint is almost equivalent to projecting the object onto a picture plane using the viewpoint as the center of the projection. In this sense, classical 3D impossible objects are not much different from their associated 2D impossible figures.

On the other hand, our new illusion is robust against shifts in viewpoint. The sense of impossibility is maintained even when we move the viewpoint slightly, or when we use our two eyes. Therefore, our new impossible object works without any strict restriction of the viewpoint, which in turn implies that it could be used as a new method for 3D art, i.e., sculptures. To extend the applicability of our objects toward this direction is one of our important future research topics.

We observe that the robustness depends on individual objects; some are very robust while others are not. At present we can tell how robust the illusion is only after we construct individual objects and check them. It is another future problem to clarify factors that guarantee the robustness of the illusion in a quantitative manner.

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