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Visualization of Pressure and Stress Distributions in Aortic Valve Simulation by Considering Heart's Pulsation and Axial Flow

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Abstract
We have performed the aortic valve simulation by considering heart's pulsation and axial flow. In the previous simulation, blood flows from the left ventricle into the aorta at a velocity changing sinusoidally. The blood in the aorta, however, flows by the heart's pulsation, which makes a different type of velocity cycle from sinusoidal wave, and also the blood flows in a swirl, which is called axial flow. The simulation has been performed with particle method, and the pressure inside the aorta as well as the stress on the aortic wall and the aortic valve have been visualized by using computer graphics techniques. As the result of the simulation, we have found that there are heavy stresses at the bottom of the aortic valve.

1 Introduction

With computer graphics and virtual reality technologies, a lot of medical systems have been developed such as preoperative surgical simulators, intraoperative navigation systems and so on. In addition, some of them employ robotic manipulators [1]. In order to simulate surgical procedures, organ models, which can be handled in computers, are necessary and they are usually generated based on 3D images, which are obtained by CT (Computed Tomography), MRI (Magnetic Resonance Imaging), MRA (Magnetic Resonance Angiography), PET (Positron Emission Tomography) and so forth. Before the model generation, target organs should be segmented in the images, and there are mainly two types of methods for segmentation: region model based level set segmentation and hierarchical segmentation. There also is a hybrid technique that uses both methods and computes the segmentation more quickly and accurately [2].

There are many organs in our body, however, blood vessel is the most important because it runs everywhere in the body and conveys nutrition to every detail parts in our body. Then, so many researches related to segmentation of blood vessel have been performed [3]. In our body, there are many kinds of blood vessels such as aorta, artery, vein, capillary and so on, and the largest and the most important one is the aorta. Then, there are lots of studies about the aorta and the heart such as aortic diameter estimation by using mono-static radar [4], heart reconstruction based on volumetric imaging [5], 3D volumetric shape reconstruction [6], and particle based blood stream simulation [7].

Inside the aorta, there is a valve called the aortic valve. If there is the aortic valvular dysfunction, the blood flows incorrectly inside the aorta, and some kinds of diseases occur. One of them is aortic valvular stenosis that narrows the aortic valvular port, and makes blood flow unsmooth so that high pressure difference occurs between the left ventricle and the aorta. Another one is aortic valvular insufficiency where the aortic valve closes incorrectly. As a result, blood flows back from the aorta to the left ventricle, and the aortic pressure lowers greatly even after the aortic valve closes [8, 9, 10, 11].

Two types of surgeries are usually performed for the above diseases. One is aortic valvular replacement (AVR), which replaces the dysfunctional live valve with an artificial one. The other is a surgery called aortic valvuloplasty (AVP), which retrieves the valvular function by repairing the dysfunctional live valve.

For the success of the surgeries, preoperative surgical simulation is mandatory since the experiment with the live aortic valve is impossible. Therefore, we have been trying a dynamic aortic valve simulation according to the blood flow inside the aorta. In addition, we have visualized the pressure inside the aorta and the stress distribution on the aortic valve [12, 13].

In our previous studies, however, the blood flows into the aorta at a velocity changing sinusoidally. In addition, the blood actually flows in a swirl by the heart's pulsation. The previous researches, however, did not consider the heart's pulsation and axial flow. Therefore, we have performed the aortic valve simulation by considering the heart's pulsation and axial flow, and also visualized the stress on the aortic wall and the aortic valve according to the simulation result [14]. In this paper, we report the visualization of the pressure distribution inside the aorta in addition to the visualization of the stress distribution on the aortic wall and the aortic valve.

2 Methods

2.1 Governing Equations

In this simulation, we have to handle two different kinds of materials: elastic body that is the aorta and the aortic valve, and fluid that is blood. Blood flow is broken by the opening and closing of the aortic valve so that the topology of the blood changes dynamically, so that it is difficult to use mesh model because mesh reconstruction is needed every time the topology changes. In addition, we cannot analyze the behavior of both elastic body and fluid with the same method if mesh model is used for elastic body and particle model is used for fluid. Then, we employ particle method for both elastic body and fluid in this simulation. There are two types of particle method: SPH (Smoothed Particle Hydrodynamics) and MPS (Moving Particle Semi-implicit) [15], and MPS can treat incompressible fluid such as blood so that we adopt MPS in the simulation. For fluid dynamics simulation, two kinds of governing equations are necessary. That is, Cauchy's equation of motion and equation of continuity, which are written as the following.

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \sigma + \mathbf{b} \tag{1}$$

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0 \tag{2}$$

Where, ρ is density, $\mathbf{v}$ is velocity, t is time, σ is stress tensor, and $\mathbf{b}$ is body force acceleration such as gravity.

In addition, constitutive equation of elastic body is described as follows.

$$\sigma^e = \lambda \mathrm{tr}\,(\varepsilon)\mathbf{I} + 2\mu\varepsilon \tag{3}$$

$$\varepsilon = \frac{1}{2}\left\{\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right\} \tag{4}$$

Where, σ^e is stress of elastic body, ε is strain tensor, $\mathbf{I}$ is unit tensor, $\mathbf{u}$ is displacement, λ and μ are lame constants, which are expressed as follows.

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \tag{5}$$

$$\mu = \frac{E}{2(1+\nu)} \tag{6}$$

Where, ν is Poisson's ratio and E is Young's module.

By substituting Eqs.(3) and (4) for Cauchy's equation (Eq.(1)), the next Cauchy-Navier equation is obtained, which equation is applied to analyze the behavior of the aortic wall and valve.

$$\rho \frac{D^2\mathbf{u}}{Dt^2} = (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u})\,\mu\nabla^2\mathbf{u} + \mathbf{b} \tag{7}$$

On the other hand, constitutive equation of fluid is written as the following.

$$\sigma^f = -p\mathbf{I} + 2\eta\mathbf{D} \tag{8}$$

$$\mathbf{D} = \frac{1}{2}\left\{\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right\} \tag{9}$$

Where, σ^f is stress of fluid, p is pressure, η is viscosity, $\mathbf{D}$ is tensor of strain velocity, and $\mathbf{v}$ is velocity. By substituting Eqs.(8) and (9) for Eq.(1), Navier-Stokes equation is obtained as follows, which is applied to analyze the behavior of blood.

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \eta\nabla^2\mathbf{v} + \mathbf{b} \tag{10}$$

2.2 Simulation Target

Fig.1 shows a simplified heart model. The most important and the largest blood vessel is the aorta, which is composed of three major parts: ascending aorta, aortic arch and descending aorta, and in which blood flows from the left ventricle to the body. In the boundary between the left ventricle and the aorta, the valve called aortic valve exists, and blood flow is controled by the opening and closing of the valve. The target of the simulation is the part which can be approximated as a cylinder from the left ventricle to the ascending aorta.

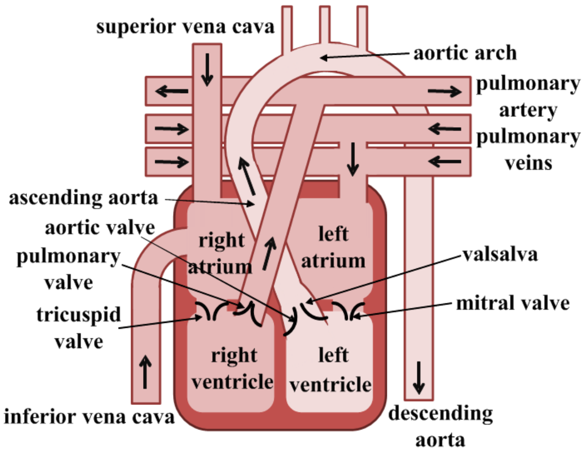


Figure 1: Simplified heart model

2.3 Heart's Pulsation

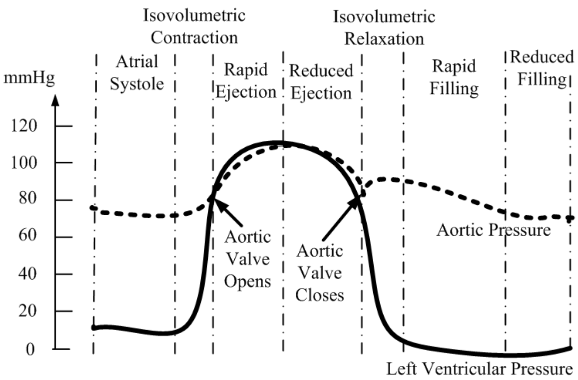


Figure 2: Diagram of heart's pulsation

Fig.2 is the diagram that shows pressure changes in the aorta and the left ventricle according to the heart's pulsation [8, 9, 10, 11]. At the phase of atrial systole, the aortic pressure is higher than the left ventricular one, and the aortic valve closes. At the phase of isovolumetric contraction, blood flows into the left ventricle and it is filled with blood. In addition, the left ventricle shrinks isovlumetrically so that the left ventricular pressure rapidly goes up to the same level as the aortic one. Actually at the phase of rapid ejection, the left ventricular pressure becomes slightly higher than the aortic one. Therefore, the aortic valve opens. While blood flows from the left ventricle to the aorta, the left ventricular pressure is almost the same level as the aortic one although the left ventricular pressure is slightly higher than the aortic one. After some blood has flown, at the phase of reduced ejection, the left ventricular pressure decreases gradually. Thus, the aortic valve closes. After the aortic valve closes, the left ventricle expands isovolumetrically at the phase of isovolumetric relaxation and the left ventricular pressure decreases rapidly. If the aortic valve closes correctly, no blood flows back from the aorta to the left ventricle and the pressure difference between the aorta and the left ventricle increases. At the phase of rapid filling, blood flows into the left ventricle so that it is filled with blood again at the phase of reduced filling.

2.4 Axial Flow

Blood flows from the left ventricle to the aorta in a swirl, which is called axial flow. Then, in order to calculate the velocity of the blood particle, the pressure diagram of Fig.2 is necessary, and the amount of flowing blood can be decided with Hagen-Poiseuille law, which is described as follows.

$$Q = \frac{\pi r^4}{8}\frac{\Delta P}{\eta L} \tag{11}$$

Where, Q is the amount of flowing blood, r is radius of blood vessel, ΔP is the pressure difference between the aorta and the left ventricle, η is coefficient of viscosity, and L is length of blood vessel. Suppose that the velocity of blood is $\mathbf{v}$, Q is rewritten as follows.

$$Q = \pi r^2 \mathbf{v} \tag{12}$$

Therefore, the vertical velocity of blood ($\mathbf{v_v}$), which is the vector along the blood vessel, can be decided as the following.

$$\mathbf{v_v} = \frac{r^2}{8}\frac{\Delta P}{\eta L} \tag{13}$$

Blood flows from the left ventricle to the aorta with axial flow, and the blood is composed of particles. Then, it is necessary to give each particle the horizontal velocity ($\mathbf{v_h}$) according to the length between the flowing central axis of the blood vessel and the position of the particle.

$$\mathbf{v_h} = l\omega \tag{14}$$

Where, l is the length between flowing central axial and particle, and ω is angular velocity of blood.

Finally, the velocity of blood becomes as follows and the simulation is performed by considering the blood velocity as each particle one.

$$\mathbf{v} = \mathbf{v_v} + \mathbf{v_h} \tag{15}$$

3 Simulation

We have performed the aortic valve simulation according to the heart's pulsation and axial flow. In the simulation, the pressure in Fig.2 is used to calculate the vertical velocity of the particle, and all materials including elastic body and fluid are composed of particles, which diameter is 2[mm]. In addition, we have used Herschel-Bulkley model [16] for the viscosity calculation of blood, and Young's modules and Poisson's ratio of elastic body were 1[MPa] and 0.4, respectively. The diameter and the length of the blood vessel were 90[mm] and 200[mm], respectively, and the coefficient of viscosity and the angular velocity were 4.7×10^{-3}[Pa • s] and 4π[rad/s], respectively.

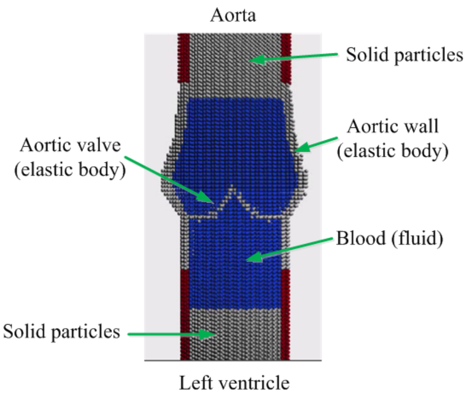


Figure 3: Simulation model of the aorta

The aortic wall and valve models were generated with computer graphics modeling software by referring to a medical book [17], and the generated model is shown in Fig.3. Blood flows from the bottom (the left ventricle) to the top (the aorta). The aortic wall and valve are composed of elastic body particles, while blood is filled with fluid particles. In the process of the simulation, solid particles, which are located at the bottom of the blood vessel, change into blood particles and flow to the left ventricle so that we become running out of solid particles. Then, solid particles, which are located at the top of the blood vessel, are forced to move to the bottom of the vessel.

The aortic wall and valve were composed with about 11k and 600 particles, respectively, while blood was composed with about 26k particles at the beginning of the simulation, and solid particles were 71k at the beginning, however, solid particles change into blood particles during the simulation so that blood partciles and solid particles were 42k and 55k at the end of the simulation for 1 cycle of the heart's pulsation.

In the simulation, a normal PC, which has i7-3770K CPU and GeForce GTX570 GPU, was used, and the response time in the simulation for 1 step, which corresponds to 0.1[ms] in real time, was about 0.2[s].

4 Results

Figs.4 and 5 show the results of the simulation. Fig.4 shows the stress on the aortic wall for the previous model that makes blood flow sinusoidally. On the other hand, Fig.5 shows the stress on the aortic wall for the proposed model that makes blood flow according to the heart's pulsation and axial flow.

By comparing Figs.4 and 5, we have found that there are higher stress on the aortic wall for the proposed model rather than the previous model, especially at the phases of rapid ejection and rapid filling. It seems that this high stress is caused by the rapid blood flow to the aorta at the phase of rapid ejection, and it is also caused by the rapid blood flow to the left ventricle at the phase of rapid filling. From Fig.2, the pressure of both the aorta and the left ventricle at the phase of rapid ejection is about 110[mmHg], which is about 15[kPa], and it corresponds to the stress on the aortic wall, which is shown in Fig.5. In addition, the pressure in the left ventricle at the phase of isovolumetric relaxation corresponds to the stress on the aortic wall in Fig.5, and the pressure in the aorta at the phase of rapid filling corresponds to the stress on the aortic wall in Fig.5. However, the pressure in the aorta at isovolumetric relaxation and that in the left ventricle at rapid filling do not correspond to the stresses on the aortic wall, which are shown in Fig.5.

On the other hand, Figs.6 and 7 show the stress on the aortic valve. From the figures, we have also found that there are higher stress on the aortic valve for the proposed model rather than the previous model, especially at the phase of rapid filling. This is due to the large amount of rapid blood flow to the left ventricle. There are heavy stresses, which particles are colored in red, at the bottom of the aortic valve.

In addition, Figs.8 and 9 show the pressure distribution inside the aorta. By comparing Figs.8(b) and 9(b), we can find that there is not so much pressure difference between the aorta and the left ventricle from Fig.9(b), compared with the result by the previous method shown in Fig.8(b). This corresponds to the diagram of heart's pulsation shown in Fig.2. The pressure value, however, is lower than that in Fig.2. It seems that the pressure value has become lower due to the calculation error of the simulation. The error is caused by mainly two following issues. One is the shape of the simulation model. The part from the left ventricle to the aorta can be approximated as a cylinder, however the part from the left atrium to the left ventricle cannot be approximated, and it affects the phases of the rapid and the reduced fillings in Fig.2.

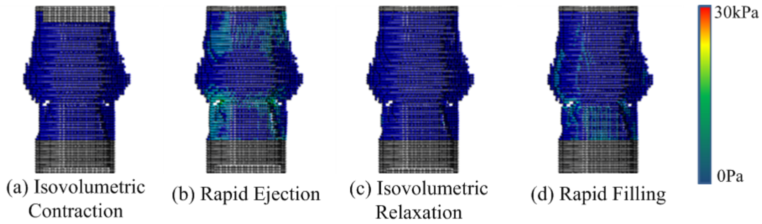


Figure 4: Stress on the aortic wall (previous model)

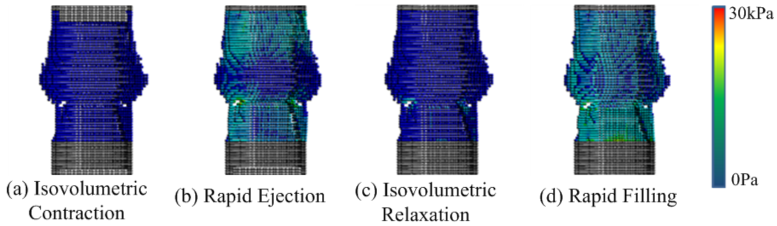


Figure 5: Stress on the aortic wall (proposed model)

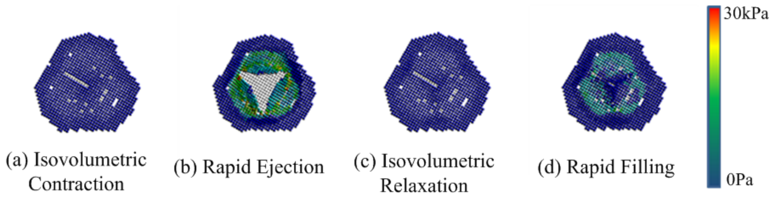


Figure 6: Stress on the aortic valve (previous model)

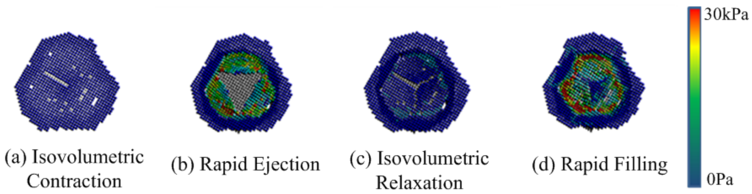


Figure 7: Stress on the aortic valve (proposed model)

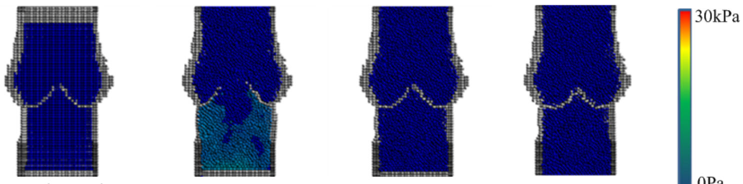
The other is the number of particles, which is limited by the calculation resources.

5 Conclusions and Discussion

We have performed the opening and closing simulation of the aortic valve according to the heart's pulsation and axial flow. The aortic wall and valve are elastic body while blood is fluid, and the topology of the blood is broken by the opening and closing of the aortic valve so that we have employed particle model for both materials. In the simulation, the velocity of the blood particle was calculated according to the pressure difference between the left ventricle and the aorta, and also axial flow. Finally, we have visualized the stress on the aortic wall and the aortic valve. As the result of the simulation, we have found that there exist higher stresses for the proposed model rather than the previous model, especially at the phase of rapid ejection and rapid filling.

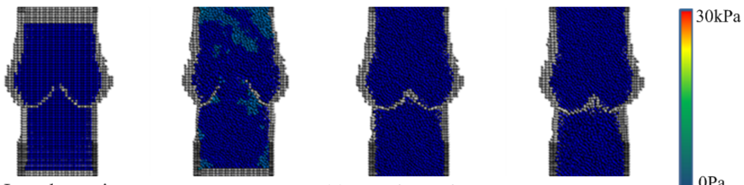
In addition, we have visualized the pressure in the aorta and the left ventricle with the result of the simulation, and have found that the pressure value has become lower than the original one shown in Fig.2. It seems that it is caused by the calculation error of the simulation.

In the future, we have to validate the proposed model; however, it is very difficult to perform the validation because we cannot measure the stress on the aortic wall and the aortic valve for the live heart. Then, we plan to validate our method by comparing the video of the live heart and the animation generated by the proposed method. We also have to analyze the calculation error of the simulation.



(a) Isovolumetric Contraction (b) Rapid Ejection (c) Isovolumetric Relaxation (d) Rapid Filling

Figure 8: Pressure inside the aorta (previous model)



(a) Isovolumetric Contraction (b) Rapid Ejection (c) Isovolumetric Relaxation (d) Rapid Filling

Figure 9: Pressure inside the aorta (proposed model)

Acknowledgments

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