Abstract
Filling holes of 3D models is an important and complicated process. In this paper, we introduce a new and robust method for filling complex holes in triangular mesh models. In order to fill separate holes, the boundary edges of a hole are smoothly arranged and then locally uniform new points are added to the hole. The new points are selected by minimizing a hole filling area. The Delaunay triangulation method in a local area is applied for creating new meshes. By considering vertices of islands in a hole, new points can be added to the hole, which allows filling of a complex hole with islands.

Keywords: 3D mesh, hole filling, point clouds.
1 Introduction

In recent years, laser scanning technology is widely used to reconstruct highly accurate digital 3D models, such as archaeological artifacts, statues, furniture and buildings. Unfortunately object complexity and self-occlusion cause scanners to miss some areas, which results in holes in a triangular mesh. Thus, filling holes is one of the important parts for 3D surface re-construction of objects.

Currently various mesh hole filling methods have been proposed. There are two kinds of hole-filling algorithms: volume-based algorithms [5, 8, 11, 15] and surface-based ones [4, 14, 18]. The basic idea of volume-based methods is to convert a surface mesh into a volume representation and to use different methods for filling holes in a volumetric space. For example, Davis et al. [8] apply a diffusion process to fill holes while Ju [11] constructs an inside/outside volume using an octree grid and reconstruct the surface by contouring it. Bischoff et al. [5] performed morphological erosion and dilation with hole boundary cells to generate a plausible partitioning. Podolak et al. [17] applied the min-cut algorithm to split a space into inside and outside portions. They converted portions containing geometric errors to an intermediate volume and made the relationship between the reconstructed surfaces and the rest of original model. Volume-based methods are robust and they work well for complex holes such as holes with islands but they are time-consuming and may generate incorrect topology in case of large holes.

The surface-based methods explicitly search holes and fill them locally on a triangular mesh. The surface-based methods generate patches to cover each hole. The surface-based methods are fast, efficient and do not change geometric information of original models. However, most of these methods do not work well on complex holes with islands. Another disadvantage of the surface-based methods is the lack of robustness. The reason is that the most surface-based algorithms are difficult to make suitable relationship between complicated outer boundaries and hole ones in curved holes.

In this paper, we introduce a new and robust surface-based hole filling method. Many kinds of holes can be created on surfaces depending on scanned point data. Filling a complex hole over an irregular region is not simple. Before filling holes on a mesh, at first the detected holes are classified into holes without islands, those with islands, and gaps.

In order to fill holes on a mesh, each hole on the surface is detected by using boundary edges. After making a convex area by arranging boundary edges of a hole, locally uniform points are added to the hole. New meshes are generated from newly added points and vertices on the boundary edges. Then Delaunay triangulation method is applied locally to create new meshes.

By considering the vertices of islands in a hole, new points can be added to the hole. If distances between new points and all vertices of the island are greater than a user-defined value, new points can be added to the hole. It allows filling of a complex hole with islands. In addition, new points are created by minimizing the hole filling area. Thus, our method can generate a continuously smooth surface on the hole area and it gets more robust when holes are filled in the mesh.

We proposed a simple technique for filling a gap. In order to fill a gap between meshes, holes are created from the gap by stitching some points on the boundary edges of the gap. The same method can be used to fill newly created holes.

Our main contributions are: (1) We propose a simple algorithm to fill complex holes with islands, (2) Our proposed method is robust for filling holes by minimizing filling areas and (3) the method can generate a continuous surface by minimizing filling area.

The rest of this paper is organized with the following sections. Section 2 introduces some surface-based hole filling methods related to our approach. Section 3 describes our method including holes and gap filling algorithms. Section 4 shows experimental results of the new method and Section 5 concludes this paper.

2 Related work

Our method belongs to the surface-based hole filling algorithms. A variety of surface-based methods have been developed for filling holes on a 3D mesh. In most of the methods, holes are identified by locating the boundary edges and the detected holes are filled by using different techniques such as minimizing hole filling areas [3], dihedral angle and Laplacian variation [14], normal variation [21], curvature variation [16], energy [6], thin-plate energy [2] and Willmore energy [7].

For example, Barequet et al. [3] applied a dynamic programming method to find a minimum area for triangulation in a 3D polygon to fill holes on a mesh. Liepa [14] introduced a new method for filling holes in unstructured triangular mesh segments by minimizing the combination of dihedral angle and the sum of triangular areas. The result-ant patching meshes interpolate the shape and density of the surrounding meshes. The method works with arbitrary holes in oriented and connected manifold meshes.

Zhao et al. [21] proposed normal variation mini-
mization technique. First, they performed interpo-
ulation of mesh segments according to the angular size of the boundaries of a hole. Starting from the smallest angle, the state of the boundaries was updated after interpolation, which was then restarted from the smallest angle. Next, the desirable normals of the new triangles are approximated using the desirable normal computing schemes. Finally, the three coordinates of the new vertex are repositioned by solving the Poisson equation based on the desirable normal and the boundary vertices of the hole.

Brunton et al. [6] proposed a novel approach to fill holes by using a minimum energy surface obtained in the following three steps. First, the hole boundaries are unfolded onto a plane using energy minimization. Second, the unfolded hole is triangulated using a constrained Delaunay triangulation. Finally, the triangular mesh is embedded as a minimum energy surface in R3. The running time of the method depends primarily on the size of the model, thereby making the method applicable to large models. Pernot et al. [16] introduced a new hole filling method by minimizing the curvature variation between the surrounding and inserted meshes.

Hu et al. [10] proposed a novel hole filling algorithm for triangular meshes. In this method, at first, a hole is triangulated into a set of new triangles using modified principle of minimum angle. Then the initial patching mesh is refined according to the density of vertices on the boundary edges. Finally, the patching mesh is optimized through the bilateral filter to recover missed features. By using experimental results, the authors showed that the proposed algorithm was robust for filling complex holes.

Wang et al. [19] proposed a hole filling method for recovering missing feature curves and corners. Feature vertices around a hole of a CAD model are extracted and classified into different feature sets. These feature sets are automatically paired, using ordered double normal, Gaussian mapping and convex/concave analysis to produce missing feature curves.

Li et al. [13] presented a feature preserving mesh hole filling algorithm. They determine incomplete feature curves around a mesh hole. They used a polynomial blending curve to restore the missing parts of the feature curves. These feature curves divide the hole into simple sub holes. Finally, a Bezier-Lagrange hybrid patch is constructed to fill each sub hole. This method is applicable for a hole with an island. Wang et al. [20] employed a prediction method that combines a hole-filling algorithm with a grey system.

All methods mentioned above have filled detected holes successfully by using different techniques. But most of these methods cannot fill holes with islands. For example, Liepa [14] and Hu et al. [10] considered a hole without islands. Otherwise the mesh would not be connected.

3 Filling holes and gaps

Gaps, holes and their islands have similar boundary structures and they can be detected by using their boundary edges. If an edge is connected with only one triangle, the edge is called a boundary edge.

A hole is a closed loop of boundary edges. A boundary vertex refers to the vertex on the boundary edges. Some complex holes can have isolated islands bounded by outer boundary edges. A gap is a narrow space between two separate hole boundaries and it can be defined as follows:

Let \(v_1, v_2, \ldots, v_n\) and \(v'_1, v'_2, \ldots, v'_m\) be closed loops of boundary edges. If a sphere with given radius centered at any vertex of the closed loop \(v_1, v_2, \ldots, v_n\) contains at least one vertex of the closed loop \(v'_1, v'_2, \ldots, v'_m\) then these closed loops create a gap.

For filling detected holes, at first, the detected holes are classified into holes without islands, those with islands, and gaps.

Figure 1 shows an overview of our hole filling algorithm. The input for the algorithm is a 3D mesh model with holes. In the first stage of the procedure, gaps are filled. Then, the boundary edges of the holes are refined by a simple algorithm. Finally, the holes are filled by adding locally uniform points.

3.1 Filling gaps

A gap usually appears on the junction between separate adjacent surfaces created from different scans or at different resolutions [12]. To fill the
gap between those surfaces, the gap on the surface is detected by using boundary edges. A gap can be filled by stitching the two sides of the gap with triangle strips. For example, Barequet [3] et al. create triangle strips that minimize the total areas of triangles and the dihedral angles from them.

In our method, all of the vertices on the gap edges are not stitched, but only those close to each other are stitched by the rule below.

First, a sphere is constructed whose center lies at a vertex on the boundary edge of the gap. An arbitrary value is given as the sphere radius. If the sphere contains any vertices lying on the other side of the gap, triangular meshes are constructed for the vertices. Implementation details are described in Figure 2. Figure 2 (a) presents a gap on the model of a fragment pot. Figure 2 (b) shows the boundary edges of the gap. Figure 2 (c) shows a sphere whose center lies on vertex $v_i$. The sphere contains vertices from two boundary edges of the gap and the triangular meshes are constructed from five vertices $v_i, v_{i-1}, v_{i+1}, v'_i, v'_{i+1}$.

The next vertex for constructing a sphere is searched within the distance given according to the constructed meshes. The process is repeated until all possible triangular meshes are constructed between the boundary edges of the gap. After this process, the gap between the boundary edges of adjacent surfaces is transformed to several separate holes. If a gap is wide, the method may fail since the sphere can contain vertices on different holes or gaps.

### 3.2 Refining boundary edges

In order to improve the quality and robustness of our method, the boundary edges of a hole are refined. In the refining stage, small angles between adjacent boundary edges are removed and concave boundary edges become almost convex. The refining procedure is similar to the hole filling method suggested by Zhao et al [21]. Boundary edges on each of the detected holes are refined as follows:

- Calculate angle $\theta_i$ between two adjacent boundary edges $(v_i, v_{i+1})$ and $(v_{i-1}, v_i)$ at every vertex point (See Figure 3).
- Starting from the vertex with the smallest angle, create new triangles on the plane determined by the following rules:
  - if angle $\theta_i$ is smaller than 75 degrees, construct a mesh $v_{i-1}, v_i, v_{i+1}$ (See Figure 3(a));
  - if angle $\theta_i$ is greater than 75 degrees and...
Repeat all steps until the process is completed.

If angle $\alpha$ is smaller than $135$ degrees, find new vertex $v_{\text{new}}$ on the plane of $v_{i-1}, v_i, v_{i+1}$ when the following conditions are satisfied (See Figure 3(b)). Here,

$$
\theta = \frac{\angle v_{i-1}v_iv_{\text{new}} + \angle v_{i+1}v_iv_{\text{new}}}{2},
$$

where $\theta_i$ is the angle of the patch.

- if angle $\theta_i$ is greater than $135$ degrees, a new mesh is not constructed.
- Update the hole by using new boundary edges
- Repeat all steps until the process is completed.

After this process, the boundary edges of the hole become almost convex. For example, Figure 4 (a) shows the boundary edges of a hole and (b) shows the result of our refining algorithm applied to the boundary edges.

By refining boundary edges, locally uniform points can be added to the hole for filling by a smooth patch. The process will be described in more detail in the following section.

### 3.3 Filling holes

In order to fill holes with almost convex boundary edges, locally uniform points are added to each of the holes. New points are added by using a similar technique introduced in [1].

In our method, new points are created by minimizing hole filling areas. Details of the method are described below.

Vertices $v_0, v_1, \ldots, v_N$ are set as the boundary vertices of a hole and edges $e_0, e_1, \ldots, e_N$ are set as the corresponding boundary edges of the hole. For boundary vertex $v_i$, boundary vertices are divided into two sets $A_i$ and $B_i$. $A_i$ is the set of the next vertices of vertex $v_i$ and $B_i$ is the set of the previous ones. The size of the two sets is equal to $\frac{N}{2} + 1$ and the sets can be formulated as follows:

$$
A_i = \{a_j | a_j = v_{i+j}, j = 0, \ldots, \frac{N}{2}\},
$$

$$
B_i = \{b_j | b_j = v_{i-j}, j = 0, \ldots, \frac{N}{2}\}.
$$

In the next step, line segments are created using vertices from sets $A_i$ and $B_i$:

$$
l_j = \{\overline{a_jb_j} | j = 1, \ldots, \frac{N}{2} - 1\}.
$$

For each boundary vertex $v_i$, a candidate vertex on the boundary is divided into two parts (in Figure 5 (a), vertex $v_3$ is divided into $a$ and $b$), and the hole filling area $S_{\text{hole},i}$ generated by line segments $\overline{a_jb_j}$ ($j = 1, \ldots, \frac{N}{2} - 1$) can be estimated as

$$
S_{\text{hole},i} = \sum_{j=0}^{\frac{N}{2}-1} S_{\Delta a_j a_{j+1} b_{j+1}} + \sum_{j=1}^{\frac{N}{2}} S_{\Delta a_j b_j b_{j+1}}.
$$

where $S_{\Delta a_j a_{j+1} b_{j+1}}$ and $S_{\Delta a_j b_j b_{j+1}}$ are areas of triangles $a_j a_{j+1} b_{j+1}$ and $a_j b_j b_{j+1}$.

In practice, most of triangles $\Delta a_j a_{j+1} b_{j+1}$ ($j = 0, \ldots, \frac{N}{2} - 1$) and $\Delta a_j b_j b_{j+1}$ ($j = 1, \ldots, \frac{N}{2} - 1$) are skinny ones and their areas can be approximated as follows:

$$
S_{\Delta a_j a_{j+1} b_{j+1}} \approx \frac{1}{2} \overline{a_j a_{j+1}} \cdot \overline{a_{j+1} b_{j+1}},
$$

$$
S_{\Delta a_j b_j b_{j+1}} \approx \frac{1}{2} \overline{b_j b_{j+1}} \cdot \overline{a_j b_j}.
$$

where $\overline{e_{av}} = \frac{1}{2} \sum_{j=0}^{N} |e_j|$ is the average length of the boundary edges of the hole. By using equations (6) and (7), the surface area $S_{\text{hole},i}$ of the patch is estimated as:

$$
S_{\text{hole},i} \approx \overline{e_{av}} \cdot \sum_{j=1}^{\frac{N}{2}-1} \overline{a_j b_j}.
$$
For example, Figure 5 (a) shows a hole with boundary vertices $v_0, v_1, \ldots, v_7$. For vertex $v_3$, boundary vertices of the hole are divided into two sets $A_4 = \{a_0, a_1, a_3, a_4\}$ and $B_4 = \{b_0, b_1, b_4\}$. Then, line segments $a_1b_1, a_2b_2$ and $a_3b_3$ are created. By Equation (8), the hole filling area that contains line segments $a_1b_1, a_2b_2$ and $a_3b_3$ is approximated by

$$S_{hole,3} \approx \overline{av} \cdot (a_1b_1 + a_2b_2 + a_3b_3).$$

(9)

For each of vertices $v_i, (i = 0, 1, \ldots, N)$ line segments are created as described above and hole filling areas $S_{hole,i}, (i = 0, 1, \ldots, N)$ are estimated by Equation (8). In order to increase robustness of our method, the vertex corresponding to the minimum hole filling area is selected for the next step.

Vertex $v_{min}$ is set as the base one to create hole filling area $min(S_{hole,i})(i = 0, 1, \ldots, N)$. For vertex $v_{min}$, line segments are created by Equation (4). New points are added for filling the hole to these line segments by the following rules:

- Find the number of division $k$ for each line segment;

$$k_j = \frac{a_jb_j}{\overline{av}} (j = 1, \ldots, N) - 1$$

(10)

where $a_jb_j$ is the length of a line segment and $\overline{av}$ is the average length of the boundary edges of the hole.

- Find points that can divide the line segment $k_j$ parts equally. Those points are considered as the candidate ones for filling the hole.

- Create a new point set for filling hole. If the sphere whose center lies on a candidate point does not contain any vertex of the object and any points of the new point set, the candidate point will be added to the set of the new points. The sphere radius $R$ is

$$R = \frac{\overline{av}}{2},$$

(11)

where $\overline{av}$ is the average length of the boundary edges of the hole.

After a new point set is created for filling the hole, meshes are constructed from points in the new point set and boundary vertices by using locally Delaunay triangulation methods such as the one suggested by Gopi et al. [9].

Figure 5 (b) shows the process for the new point addition. Vertex $v_2$ is the base one to create the minimum hole filling area. Line segments are created for this vertex. Then, red dots, which are the candidate points, are created on the line segments. Since sphere with center point $p$ does not contain any vertex or blue dot, center point $p$ will be added to the set of new points. Since sphere with center point $q$ contains boundary vertex $v_6$, point $q$ is not added to the set of new points. Figure 5 (c) shows the final result of the hole filling process.

### 3.4 Filling holes with islands

To fill a hole with islands is one of the important parts of the hole filling process. Islands in a hole can be detected from the boundary edges. An island is bounded by the outer boundary edges.

After an island is detected from the outer boundary edges, a hole containing the island is identified as follows.

Island $I$ is set from boundary vertices $\overline{v_i}, \overline{v_j}, \ldots, \overline{v_N}$ and holes $H_0, H_1, \ldots, H_M$ are detected on a mesh. Then, distance $d_{I,H_j}$ between island $I$ and hole $H_j$ is defined as

$$d_{I,H_j} = \frac{\sum_{i=1}^{N} d(\overline{v_i}, \overline{v_i}')}{N},$$

(12)

where $\overline{v_i}'$ is the closest boundary vertex from hole $H_j$ to vertex $\overline{v_i}$ and $d(\overline{v_i}, \overline{v_i}')$ is the distance between vertices $\overline{v_i}$ and $\overline{v_i}'$.

Hole $H$ with distance

$$min(d_{I,H_j}) \ (j = 0, \ldots, M),$$

(13)
can contain island $I$.

For filling a hole with islands, new points should be considered before adding the points to the hole. If a sphere whose center lies on a candidate point contains any vertex of an island, the point cannot be added to the hole. After creating a new point in the hole, meshes are constructed from newly added points, boundary vertices of the hole and the island.

Figure 6 (a) shows a hole with an island and (b) shows the result of the hole filling algorithm. The orange lines represent the new mesh.

If the distances between newly added points and vertices of an island are greater than the sphere radius defined by Equation (11), the new method cannot fill the hole with the island. This is the disadvantage of our algorithm.

4 Experimental results

This section describes some results of our approach for filling holes. We have implemented our algorithm on various 3D models, such as archaeological fragments. All objects have many kinds of complex holes.

The first experiment was implemented on a 3D fragment model of a pot shown in Figure 7 (a). The model consists of two separate surfaces, caused by scanning with different devices. The gap between the two surfaces was stitched by our method as shown in Figure 2. After stitching, the gap was transformed to several separate holes. Then the holes were filled by using our method.

In Figure 7 (a), the blue triangular meshes of the object are shown. The boundary edges of the hole are indicated by red lines. Figure 7 (b) shows the result of our hole filling algorithm. In this experiment, the sphere radius is set with the doubled value of the average distance of points. The 3D fragment model has 48,563 vertices and 95,822 triangular faces. After filling the hole, the model has 50,298 vertices and 99,368 triangular faces.

In the next experiment, a hole on a rabbit model was filled by our new algorithm. Figure 8 (a) shows the model with a hole. The result without area minimization is presented in Figure 8 (b). In this case, for an arbitrary vertex among the boundary vertices, new points were added by creating line segments and then the hole was filled. Figure 8 (c) shows the result of our algorithm when the area was minimized.

Figure 9 (a) shows the 3D model of a stone tool. The surface of the model contains a hole with an island. The closed boundary edges of the hole and the island are indicated by bold red lines. The hole was filled by adding new points with consideration of vertices of the island. The result is shown in Figure 9 (b).

Figures 10 and 11 show the results of the hole filling algorithm applied to different 3D models of earthenware objects. The 3D model in Figure 10 has 227,030 vertices and 450,700 triangular faces while the model in Figure 11 has 642,570 vertices and 1,276,560 triangular faces. Some holes on models contain an island such as shown in Figure 6.
Figure 8: (a) rabbit model with a hole, (b) result without hole filling area minimization, (c) result with hole filling area minimization.

After filling the holes, the model in Figure 10 has 236,950 vertices and 472,410 triangular faces, and the model in Figure 11 has 667,314 vertices and 1,332,850 faces.

From experimental results, the proposed method is fast and easy to implement. The computation time for filling each hole is less than 5 seconds on an Intel Core i5 processor. However, our method can not construct continuously smooth surface on a hole on a highly curved area because holes are filled by nearly planner surface.

5 Conclusion

Filling holes on a 3D model is an important and complicated process. Due to several reasons, there are many kinds of complex holes on the surface of an object. In this paper, we discussed new hole filling algorithm of 3D meshes. In order to fill holes, boundary edges of holes are redefined and new points are added to the hole by minimizing hole filling areas. Locally Delaunay triangulation method is applied to construct new meshes for filling holes. For filling complex holes such as those with islands, new points are added to the holes by considering vertices of the islands. Before using this algorithm, gaps of objects are converted into separate holes by stitching the vertices on the boundary edges of the gap.

Our experimental results showed that the proposed method is robust for filling holes on meshes and effective for filling holes with islands.

References

Figure 11: (a) 3D model with a hole, (b) result of the hole filling algorithm. (The data is provided by Tokyo National Museum.)


